KHAITAN PUBLIC SCHOOL, SAHIBABAD
HOLIDAY HOME WORK-2017
CLASS – XII

MATHEMATICS

1. Let R be a relation on **R,** defined by $R = \{(a,b): a^2 + b^2 = 1\}$. Show that R is symmetric but neither reflexive nor transitive.
2. Show that the relation R defined by $(a,b)R(c,d) \Rightarrow a + d = b + c$ on $A \times A$, where $A =: 1,2,3,4 \dots \dots .. 10\}$ is an equivalence relation. Hence write the equivalence class of $[(3,4)];\ a,b,c,d \in A$
3. Let R be a relation on $N \times N$, defined by $(a,b)R(c,d) \Leftrightarrow ad(b + c) = bc(a + d)$. Check whether R is an equivalence relation or not. (CBSE,2015)
4. Let R be a relation defined on N as $R = \{(x,y): 2x + y = 41; x,y \in N\}$. Find the domain and range of R. Also verify whether R is reflexive, symmetric, transitive.
5. Let R be a relation defined on $N \times N$ by $(a,b)R(c,d) \Leftrightarrow ab(b + c) = bc(a + b)$. Show that R is an equivalence relation.
6. If the function $f: \boldsymbol{R} \to \boldsymbol{R}$ defined by $f(x) = 2x - 3$ and $g: \boldsymbol{R} \to \boldsymbol{R}$ defined by $g(x) = x^3 + 5$, then find the value of $(fog)^{-1}$.
7. Determine whether the function $*$ on $N \times N$ is defined by $(a,b) * (c,d) = (ac, bd)$ is a binary operation or not. If yes, determine whether it is commutative and associative. Find the identity(if exists). Find all the invertible elements (if any) and their inverses. (Exemplar)
8. Show that $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}; x \in R$ is neither one-one nor onto.
9. Let $*$ be a binary operation defined on $Q \times Q$ by $(a,b) * (c,d) = (ac, b + ad)$, where Q is the set of rational numbers. Determine, whether $*$ is commutative and associative. Find the identity element for $*$ and the invertible elements of $Q \times Q$. (CBSE-SQP,2017)
10. On the set $M = A(x) = \left\{\begin{bmatrix} x & x \\ x & x \end{bmatrix} : x \in R\right\}$ of 2 x 2 matrices, find the identity element for the multiplication of matrices as binary operation. Also find the inverse of an element of M.
11. If the function $f: [1, \infty) \to [1, \infty)$ defined by $f(x) = 2^{x(x-1)}$ is invertible, find $f^{-1}(x)$.
12. Find the value of the expression: $\sin[cot^{-1}(\cos\{tan^{-1}1\})]$. (Exemplar)
13. Find the value of $tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$ (Exemplar)
14. Prove that : $2tan^{-1}\left(\frac{1}{5}\right) + sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$ (CBSE, 2014)
15. Prove : $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2 \ , x^2 \leq 1$ (C.B.S.E. 2015)
16. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, xy<1, then write the value of $x + y + xy$. (C.B.S.E. 2014)
17. Find the value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$ (C.B.S.E. 2011)
18. Solve for x: $\sin[\cot^{-1}(x + 1)] = \cos(\tan^{-1} x)$ (C.B.S.E. 2015)
19. Solve for x: $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$
20. Solve for x: $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$ (C.B.S.E. 2015)
21. Solve for x: $sin^{-1}6x + sin^{-1}6\sqrt{3}x = \frac{\pi}{2}$
22. Solve for x: $\tan^{-1}\frac{x+1}{x-1} + \tan^{-1}\frac{x-1}{x} = \tan^{-1}(-7)$

23. Without finding (adj. A), find A(adj. A) when A=$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 0 & 1 \end{pmatrix}$

24. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

25. Without expanding the determinant at any stage, prove that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a,b,c are in A.P.

26. Using properties of determinants prove that:

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$$ **(HOTS)**

27. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}, B\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB. Hence, use it to solve the following system of equations:

$x - 2y = 10$

$2x + y + 3z = 8$

$-2y + z = 7$

28. Given that $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Find AB. Use this to solve that following system of equations : $x - y = 3,\ 2x + 3y + 4z = 17,\ y + 2z = 7$ **(CBSE, 2017)**

29. Using properties of determinants, prove that:

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & z & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

30. Using properties of determinants, prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by $(x+y+z)$, and hence find the quotient. (CBSE, 2016)

31. If x, y, z are all different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, prove that:

$x^{-1} + y^{-1} + z^{-1} = -1$ (CBSE, 2016)

32. Find the relationship between 'a' and 'b' so that the function 'f' defined by :

$$\mathrm{f(x)} = \begin{cases} \mathrm{ax+b} & ,\mathrm{if\ x} \le 3 \\ \mathrm{bx+3} & ,\mathrm{if\ x} > 3 \end{cases}$$ is continuous at x=3

33. Find the value of k so that the function 'f' defined by $\mathrm{f(x)} = \begin{cases} \mathrm{kx+1}, & \mathrm{if\ x} \le \pi \\ \cos x, & \mathrm{if\ x} > \pi \end{cases}$ is continuous at $\mathrm{x} = \pi$

34. Find 'a' so that $f(x) = \begin{cases} a\sin\left[\frac{\pi}{2}(x+1)\right], & if\ x \le 0 \\ \frac{\tan x - \sin x}{x^2} & , if\ x > 0 \end{cases}$

Is continuous at x=0.

35. Find the value of 'a' and 'b' m for which $f(x) = \begin{cases} \frac{1-sin^2 x}{3cos^2 x}, if\ x < \frac{\pi}{2} \\ \text{a} \quad , \text{if } x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}\ ,\ if\ x > \frac{\pi}{2} \end{cases}$

Is continuous at $x = \frac{\pi}{2}$.

36. Differentiate $\tan^{-1}\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\}$ w.r.t. $\cos^{-1}x^2$.

37. Differentiate $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ with respect to $\sin^{-1}(2x\sqrt{1-x^2})$.

38. If y=$[x+\sqrt{x^2+a^2}]^n$, prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$

39. If y=log$\sqrt{\frac{1+\tan x}{1-\tan x}}$, prove that : $\frac{dy}{dx} = \sec 2x$

40. If $y = sin^{-1}\left[\frac{12x+5\sqrt{1-x^2}}{13}\right]$, find $\frac{dy}{dx}$.

41. If $\sqrt{1-x^6+}\sqrt{1-y^6} = a(x^3-y^3)$, $\frac{dy}{dx} = \frac{x^2\sqrt{1-y^6}}{y^2\sqrt{1-x^6}}$

42. If y$\sqrt{1-x^2}+x\sqrt{1-y^2} = 1$, prove that: $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

43. If $\sin y = x\ sin(a+y)$, prove that: $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

44. If $x^y = e^{x-y}$, prove that: $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

45. If $a^x + a^y = a^{x+y}\ (a>0)$, prove that $\frac{dy}{dx} + a^{y-x} = 0$

46. If $(x+y)^{m+n} = x^m y^n$,prove that: $\frac{dy}{dx} = \frac{y}{x}$

47. If $e^y = y^x$, P. T. $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$

48. Given that $\cos\frac{x}{2}.\cos\frac{x}{2^2}.\cos\frac{x}{2^3}\ldots\ldots\ldots. = \frac{\sin x}{x}$ prove that

$\frac{1}{2^2}sec^2\frac{x}{2} + \frac{1}{2^4}sec^2\frac{x}{2^4} + \frac{1}{2^6}sec^2\frac{x}{2} + \cdots\ldots. = cosec^2 x - \frac{1}{x^2}$

49. Differentiate $y = x^{(x^x)} + (x^x)^x$ w.r.t. x.

50. If $x = \frac{1+t}{t^3}$, $y = \frac{3}{2t^2} + \frac{2}{t}$, show that $\frac{d}{dx}\left[\frac{1+\frac{dy}{dx}}{\left(\frac{dy}{dx}\right)^3}\right] = 1$