

MATHEMATICS

- Let  $P = \{\frac{1}{x} : x \in N, x < 7\}$  and  $Q = \{\frac{1}{2x} : x \in N, x \leq 4\}$ . Find  $Q \cup P$ .
- Let  $U = \{x : x \leq 10, x \in N\}$ . If  $A = \{x : x \in N, x \text{ is prime}\}$  and  $B = \{x : x \in N, x \text{ is even}\}$ . Write  $A - B'$  in roaster form.
- Represent  $A \cup B'$  and  $A \cap B'$  using venn diagram.
- In a certain town 25% families own a phone, 15% own a car, 65% own neither a phone nor a car, 2000 families own both the phone and a car. Find (i) How many families live in the town? (ii) How many families own either a phone or a car.
- A survey shows that 63% of Americans like cheese, 76% of them like apples. If  $x\%$  likes both cheese and apples, then find 'x'.
- A survey of 500 television viewers produced the given information: 285 watch football, 195 watch hockey, 115 watch cricket, 45 watch football and cricket, 70 watch football and hockey, 50 watch cricket and hockey, 50 do not watch any of the three games. How many watch exactly one of the three games?
- If  $(a, -2)$  &  $(4, b^2)$  belongs to the relation  $R$  on  $Z$  where  $R = \{(x, y) : x, y \in Z \text{ \& } y = 2x - 4\}$ , then find the values of  $a$  &  $b$ .
- Express the relation in Roaster form:  $R = \{(x, y) : y + 2x = 5, x, y \in W\}$
- Let  $A = \{3, 5\}$  and  $B = \{7, 11\}$ . Let  $R = \{(a, b) : a \in A, b \in B, a - b \text{ is odd}\}$ . Show that  $R$  is an empty relation from  $A$  to  $B$ .
- If  $A = \{1, 2, 3\}$ ,  $B = \{4, 5\}$  and  $C = \{5, 6\}$ , then verify that :  $A \times$   
 $(B - C) = (A \times B) - (A \times C)$
- If  $A \times B = \{(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)\}$ , find  $B \times A$ .
- Find the range of the following functions (i)  $f(x) = x^2 + 3$  (ii)  $g(x) = \frac{2x+1}{3x-2}$
- Write the domain of  $f(x) = \frac{1}{\sqrt{x-4}} + \sqrt{x-3}$
- Find the domain of  $f(x) = \frac{x+7}{x^2-8x+4}$
- Write the domain and range of  $f(x) = \sqrt{16-x^2}$
- If  $f(x) = \sqrt{x-1}$  and  $g(x) = 3 - 2x$ , find the value of (i)  $3f-2g$  (ii)  $2f^2+3g$
- If  $\sin \alpha + \sin \beta = a$  and  $\cos \alpha + \cos \beta = b$ , show that : (i)  $\cos(\alpha + \beta) = \frac{b^2-a^2}{b^2+a^2}$   
(ii)  $\sin(\alpha + \beta) = \frac{2ab}{b^2+a^2}$
- If  $\cos(\theta + 2\alpha) = m \cos \theta$ , show that,  $\cot \alpha = \frac{1+m}{1-m} \tan(\theta + \alpha)$ .
- Prove that :  $\frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A$
- $\sin A \sin(60^\circ - A) \sin(60^\circ + A) = \frac{1}{4} \sin 3A$

21. Solve :  $\sin 2x - \sin 4x + \sin 6x = 0$ .

22. Solve : (i)  $4\cos^2x + 6\sin^2x = 5$

(ii)  $\cos 3A + 8\cos^3 A = 0$

(iii)  $4 \sin x \cdot \sin 2x \cdot \sin 4x = \sin 3x$

(iv)  $\sin 2x + \sin 4x + \sin 6x = 0$

23. Prove that :  $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$

24. Prove that :  $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

25. Prove that :  $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$

26. Prove that :  $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

27. Prove that :  $\cos^2 A + \cos^2 \left( A - \frac{2\pi}{3} \right) + \cos^2 \left( A + \frac{2\pi}{3} \right) = \frac{3}{2}$

28. Prove :  $\cos \left( \frac{3\pi}{4} + x \right) - \cos \left( \frac{3\pi}{4} - x \right) = -\sqrt{2} \sin x$ .

29. Prove:  $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta \cos 4\theta$

30. Show that :  $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8\theta}}} = 2 \cos \theta$

31. Prove that :  $\cos \alpha + \cos \beta + \cos \gamma + \cos(\alpha + \beta + \gamma) = 4 \cos \frac{\alpha+\beta}{2} \cos \frac{\beta+\gamma}{2} \cos \frac{\gamma+\alpha}{2}$

32. If x and y are two different integers then using PMI prove that  $x^n - y^n$  is divisible by (x-y)

33. Using PMI prove :  $\left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \dots \dots \dots \left(1 - \frac{1}{n+1}\right) = \left(\frac{1}{n+1}\right)$

34. Prove by using principle of mathematical induction, for all  $n \in N$ :

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots \dots \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{n+1}$$

35. Prove by using Principle of mathematical induction for all  $n \in N$ ,

$$1.2 + 2.2^2 + 3.2^3 + \dots \dots \dots + n.2^n = (n-1)2^{n+1} + 2$$

36. Prove by using P.M.I.  $10^n + 3.4^n + 5$  is divisible by 9, for all  $n \in N$

37. Find the values of x and y, if  $\frac{1+ix}{3+i} - \frac{2-iy}{3-i} = i$

38. For any two complex numbers  $z_1, z_2$ , prove that  $\overline{z_1 z_2} = \overline{z_1} \overline{z_2}$ . (Take  $z_1 = a + ib, z_2 = c + id$ .)

39. Evaluate:  $(2 + i^3) - (4 - i^5) + (7 + 8i^2)$

40. Find the value of x and y if  $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$